Experimental determination of the speed of light

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By exploiting the resonance condition of an open cavity He-Ne laser, we measure the variation between longitudinal mode frequencies, or beat frequencies, as a function of cavity length. Using a high frequency photodiode and radio spectrum analyzer, we empirically determine the speed of light as \( c = 299.55(7) \times 10^6 \text{ m/s} \) - within 0.79% of the defined value. The measurement of the speed of light is fundamental to the realization of both a physical constant – the speed at which electromagnetic radiation propagates - and the standard unit of length from which it is defined, the meter.

Introduction

The Helium Neon laser emits a nearly perfect monochromatic and coherent beam of electromagnetic radiation at a wavelength of 632 nm. However, fundamental limits exist as to just how monochromatic this light can be.

Quantum mechanics imposes an inherent energy-time indeterminacy\(^{1}\) from energy level transitions in a four level laser system while relativistic effects introduce Doppler shifting\(^{2}\), but these effects are minimal when compared to the resonant coupling of the axial modes to higher-order modes\(^{3}\). This experiment exploits these mode frequency degeneracies and the optical cavity resonance condition to empirically determine the speed of light.

These effects would otherwise result in a continuous spread in wavelength from the exit aperture of the optical cavity if it were not for the optical cavity resonance condition, which effectively discretizes the continuous spread to allow only certain frequencies, the longitudinal modes of the cavity. Thus, the frequency \( (v) \) depends on the cavity length \( (L) \) and speed of light \( (c) \) in the integer step \( (k) \) condition:

\[
v = \frac{kc}{2L}
\]

To measure the difference in frequencies, \( \Delta v \), or the beat frequency between neighboring longitudinal modes, we use a high speed (up to 1.5 GHz) photodiode connected to a radio spectrum analyzer. This frequency difference, the free spectral range, is directly related to the time it takes light to complete one oscillation in the cavity by the above expression. However, the oscillation time is not purely a function of the cavity length and speed of propagation. Indeed, the cavity length traversed within the resonator travels through a gain medium. Thus, the path length is dependent on the refractive index of the gain medium, \( n \), as it represents the net optical path:

\[
v = \frac{kc}{2nL}
\]

It can be shown\(^{4}\) that close to a resonance transition, the refractive index exhibits complex frequency dependent behavior. Thus, to minimize the effects of the varying refractive index, we observe how the free spectral range varies as small adjustments are made to cavity length \( (\Delta L) \) using a micrometer. Plotting the inverse of the free spectral range as a function of the change in cavity length is used to empirically determine the speed of light by linear regression.

Experimental Methods

The laboratory setup depicted in Fig. 1 centers around a He-Ne discharge tube in a Fabry-Perot type optical resonator. The laser is aligned by adjustment of the two mirrors to achieve a lasing state and a maximal power output of \( 18 \pm 2 \text{ \mu W} \) as measured by a laser power meter. An aperture is inserted in the optical cavity in front of the right mirror and closed such that the resulting beam matches the intensity distribution of the TEM\(_{00}\) mode (see Table 1).

![Figure 1 - Laboratory setup](image)

The output beam is directed by a mirror through a lens that focuses the beam onto a high
frequency (1.5 GHz max) photodiode. The photodiode output is observed on a radio spectrum analyzer in the 300 MHz region. Our spectrum analyzer performed optimally, at ±0.001 MHz resolution, when averaging over 10 sweep intervals of 135 milliseconds each with a resolution bandwidth of 1 kHz and a video bandwidth of 30 Hz.

Once a clear signal is ascertained, incremental adjustments are made to the optical cavity length by shifting the left mirror using a micrometer (accurate to ±2.54 µm). The experiment was conducted a total of six times with over 100 data points in the TEM$_{00}$ mode for the final determination of $c$.

Lastly, we observed a mode dependent frequency pulling and pushing as a function of the iris diameter. To quantify this potential source of error, we project the laser beam onto a distant surface and count the number of nodes present to determine the TEM mode. Once a stable TEM mode is acquired, we repeat the experiment to determine the shift in the reference (TEM$_{00}$ mode) free spectral range.

**Results and Data Analysis**

In the TEM$_{00}$ mode, we first measured the initial beat frequency at $\Delta L = 0$ as $v_{00} = 300 \pm 0.003$ MHz. The inverse of the observed beat frequencies as a result of varying the optical cavity from 0 to 3 cm is displayed in Fig. 2.

![Figure 2 - Inverse free spectral range as a function of cavity length](image)

Linear regression was performed using the least squares method for the total data set of 100 points and yielded a value of $c = 299.55(7) \times 10^6$ m/s. Compared to the defined value of $299792458$ m/s, our measurement is accurate to within 0.79 %. Further analysis of the linear curve fit residuals indicate the possibility of additional sources of systematic errors as the distribution appears to have a small bias (Fig. 2). We believe that this can be explained by three leading contributions to the error in our measurement - from the angular misalignment of the micrometer guiderail relative to the optical axis, the micrometer tolerance and changes in reference free spectral range caused by the presence of higher order longitudinal modes. Statistical errors were on the order of 0.02 % relative to $c$.

![Figure 3 - Residuals plot from linear curve fit](image)

As a function of TEM mode, we noticed fluctuations in beat frequency when compared to the TEM$_{00}$ mode (Table 1). By conducting the experiment in different modes, it became clear that small beat frequency fluctuations yielded another 0.4% inaccuracy in determining the speed of light and could explain the error in our initial value. The observed increase in the reference free spectral range would tend to lower the measured value of $c$ by a maximum of 0.4 %.

Lastly, we could not explain a slight asymmetry to the observed beat frequency peaks on the spectrum analyzer. In particular, the peaks appeared to have a slight tail stretching into the higher frequencies, but we believe that this effect was of little consequence to our final values and its effect was minimized by averaging over 10 sweep cycles.
Conclusions

By measuring how the free spectral range changes as a function of optical cavity length in a He-Ne laser, we empirically determined the speed of light to within 0.79 % of the defined value as \( c = 299.55(7) \times 10^6 \) m/s. Based on quantitative analyses, we believe that frequency pulling from other TEM modes as well as angular errors in the optical cavity alignment were the primary sources of error in our measurement.

References